



Muon Collider Lattice Design

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Design Goals

- High Luminosity (Higgs Factory $L \sim 10^{32} \text{cm}^{-2}\text{s}^{-1}$, 3TeV MC $L > 4 \cdot 10^{34} \text{cm}^{-2}\text{s}^{-1}$)
 - \Rightarrow round beams (to minimize beam-beam effect)
 - \Rightarrow small β^* (Higgs Factory $\beta^* \sim 2 \div 3 \text{ cm}$, 3TeV MC $\beta^* \sim 3 \div 5 \text{ mm}$)
 - \Rightarrow small circumference
 - \Rightarrow small bunch length $\sigma_s \leq \beta^*$ (high-energy MC)
 - \rightarrow momentum compaction factor $\sim 10^{-5}$
- Acceptable detector backgrounds
 - \Rightarrow tight apertures in W absorbers (resistive wall instability?)
 - \Rightarrow dipole component in FF quads
 - \Rightarrow halo extraction (bent crystals?)
- Manageable heat loads in magnets
 - \Rightarrow enough space for W absorbers, shorter distance between masks
- β^* variation in wide range (w/o breaking dispersion closure)
- Small collision energy spread $\sigma_E/E \leq 4 \cdot 10^{-5}$ (for Higgs Factory)
 - \Rightarrow instabilities? longitudinal beam-beam effect?
- Safe levels of ν -induced radiation (for $E \geq 3 \text{ TeV}$)
 - \Rightarrow no long straights (except for IRs)
 - \Rightarrow combined-function magnets to spread ν 's

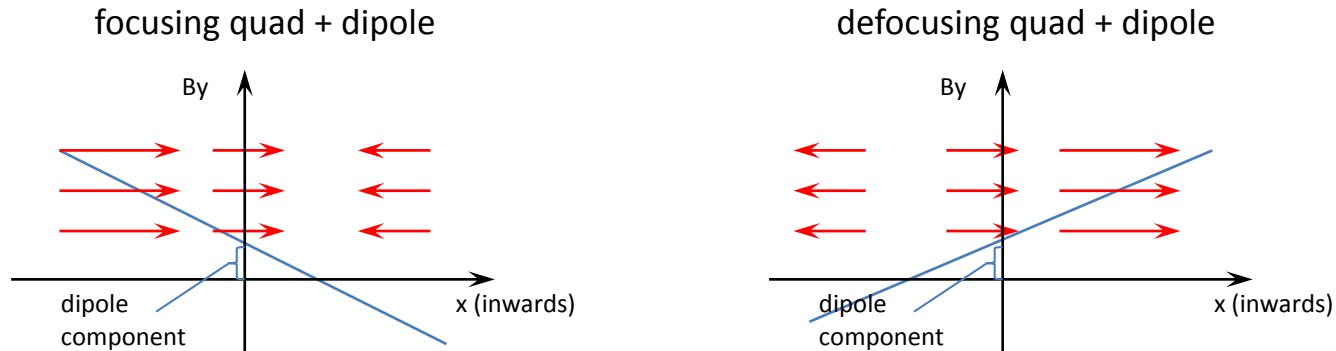
Basic Concepts

In the course of different versions of the Muon Collider (Higgs Factory, 1.5TeV, 3TeV) new solutions were found, two of them (IR chromaticity correction scheme and arccell design) can find application in machines other than MC:

- **Quadruplet Final Focus**
 - ⇒ better detector protection from secondaries than with a triplet FF
- **3-sextupole chromaticity correction scheme**
 - ⇒ 1st sextupole from IP corrects vertical chromaticity while 2nd and 3rd sextupoles form -/ separated pair for horizontal correction
- **New Flexible Momentum Compaction arccell design**
 - ⇒ (large) negative momentum compaction factor, independent control of tunes, chromaticities, momentum compaction factor and its derivative with momentum
- **β^* -tuning section with a chicane**
 - ⇒ allows for β^* variation in a wide range and has bending field everywhere to spread decay v's

All these solutions were incorporated in the latest 3TeV collider design

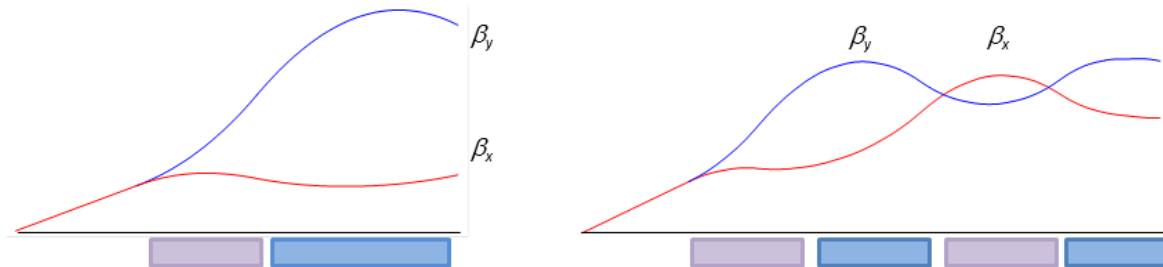
Why Quadruplet Final Focus?



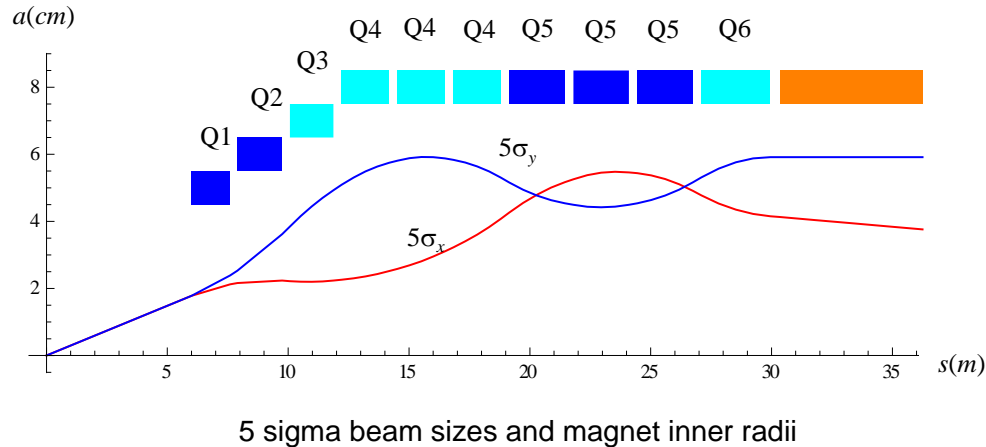
- Dipole component in a defocusing quad is more efficient for cleaning purposes – it is beneficial to have the 2nd from IP quad defocusing
- The last quad of the FF “telescope” also must be defocusing to limit the dispersion “invariant” generated by the subsequent dipole (not shown)

$$J_x = \frac{D_x^2 + (\beta_x D'_x + \alpha_x D_x)^2}{\beta_x} \approx \beta_x \phi^2$$

– both requirement are met with either doublet or quadrupole FF:



Quadruplet Final Focus



	Q1	Q2	Q3	Q4	Q5	Q6
aperture (mm)	90	110	130	150	150	150
G (T/m)	267	218	-154	-133	129	-128
B ₀ (T)	0	0	2	2	0	2
B _{pole tip} (T)	12.0	12.0	12.0	12.0	9.7	11.6
length (m)	1.6	1.85	1.8	1.96	2.3	2.85

Parameters of the Final Focus quadrupoles

Quad inner radii satisfy requirement

$$R > 5 \sigma_{\max} + 2 \text{ cm}$$

which guarantees that the beam will be in a good field region and provides enough space for absorber.

The maximum pole tip field was increased up to 12 T. If this is not feasible, the apertures can be reduced: we do not need 5σ for the beam scraped at 3σ .

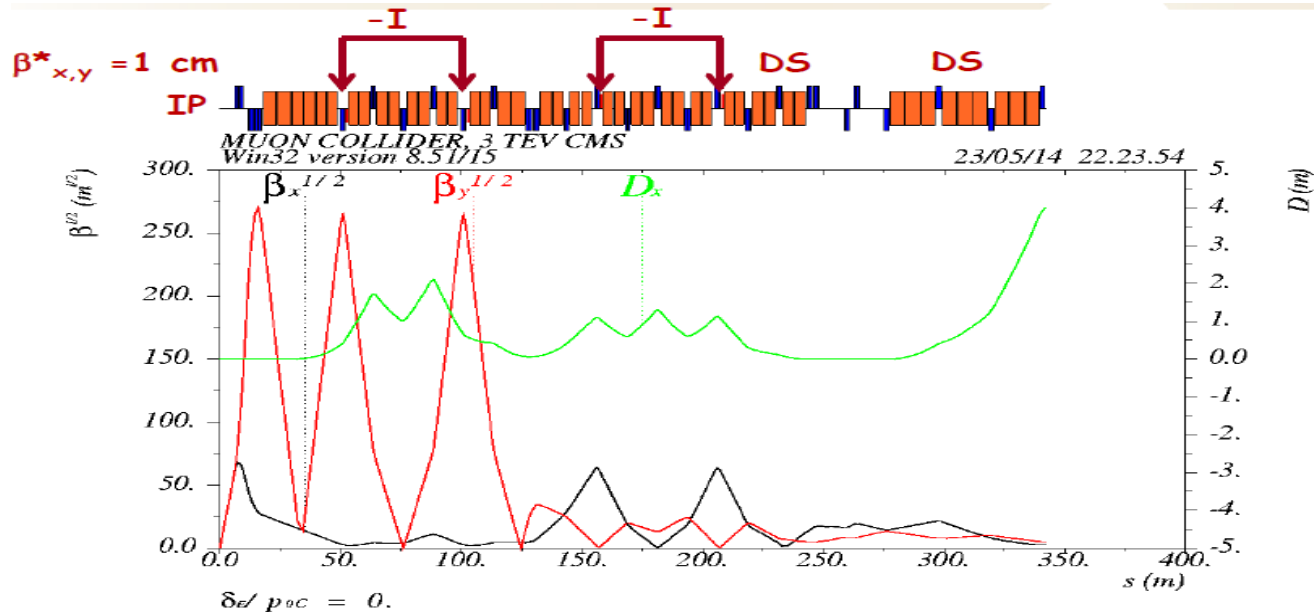
Maximum magnet aperture is noticeably reduced – 150mm vs 180mm – compared to the previous design based on a triplet FF and 10T pole tip field .

A drawback of the quadruplet FF: high β_x in IR dipoles

Chromaticity Correction

Very popular (but not yet realized) is the scheme with two $-I$ blocks (J.Irwin et al., 1991). It can be called “4-sextupole scheme”.

The latest example: 3TeV MC design developed at SLAC (M.-H. Wei et al.)

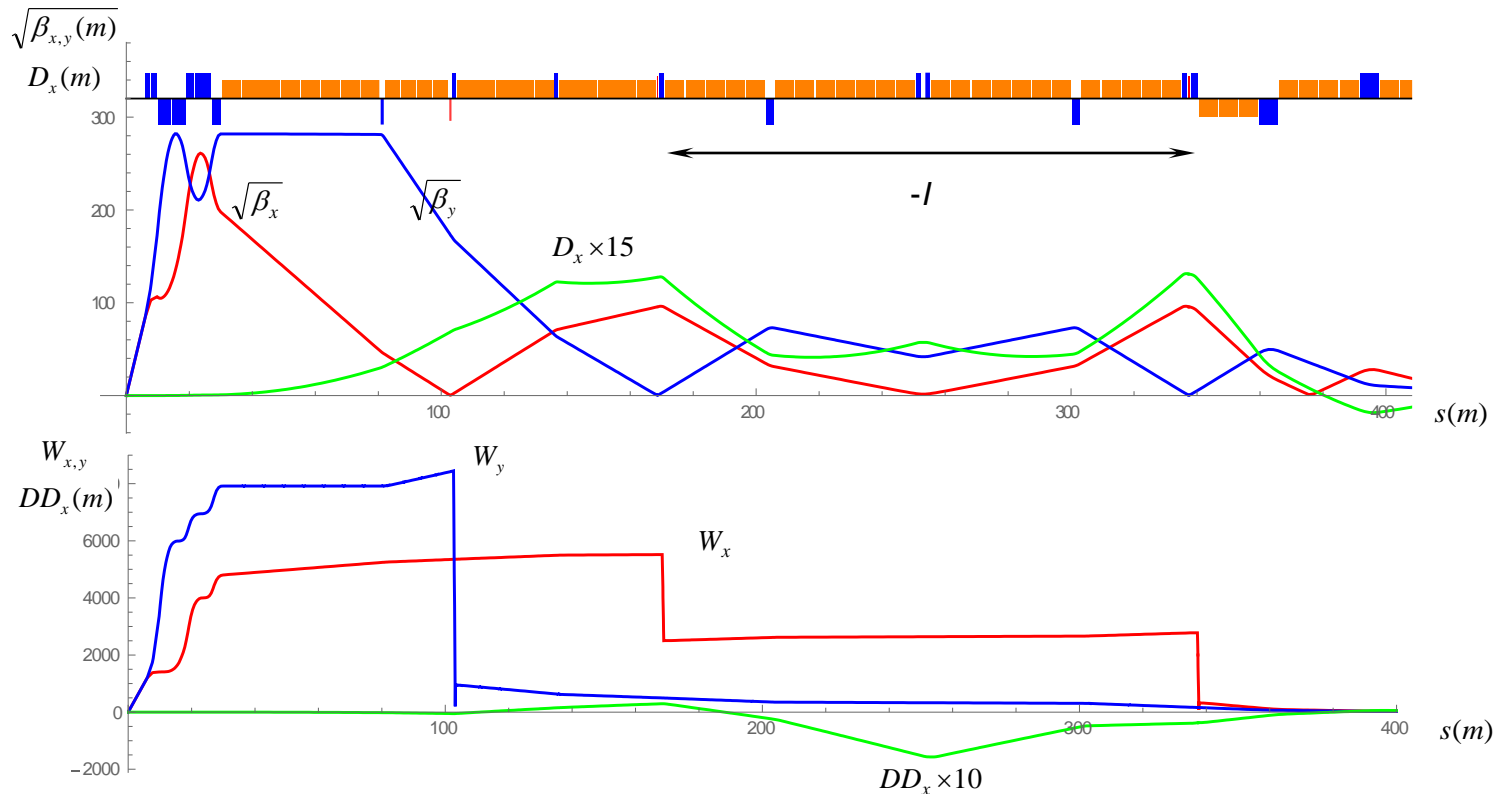


Issues with the 4-sextupole scheme:

- $-I$ blocks themselves produce significant contribution to chromaticity
- There is a strong uncompensated nonlinearity in centrifugal force \rightarrow adverse effect on DA
- Many elements at high-beta locations \rightarrow high sensitivity to errors
- Large positive contribution to the momentum compaction factor \rightarrow a strain on the arc lattice which must compensate it

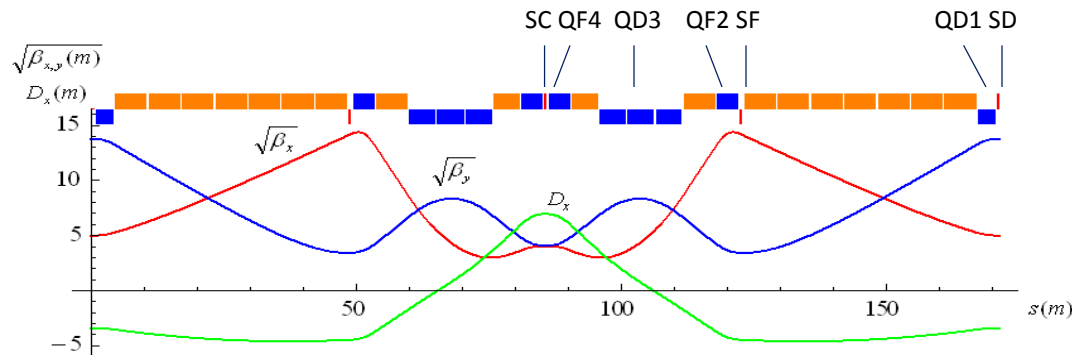
Chromaticity Correction

To address the above-mentioned issues a “3-sextupole scheme” was developed at FNAL. It uses just one sextupole (at each side of IP) for vertical chromaticity correction relying on small β_x for aberration suppression.



Optical (top) and chromatic (bottom) functions at IR and chromaticity correction section

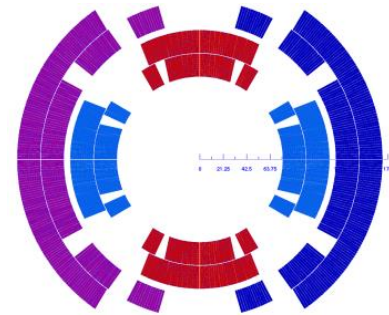
Arc Cell with Combined Function Magnets



Motivation:

- Spread decay v's
- Sweep away decay electrons before they depart from median plane - allows for azimuthally tapered absorber

Magnet	L(m)	G(T/m)	B(T)	$4\sigma_x(\text{cm})$	$4\sigma_y(\text{cm})$
QD1	3.34	-31	9	1.41	0.23
QF2	4	85	8	1.80	0.07
QD3	5	-35	9	1.43	0.14
QF4	4	85	8	2.80	0.08



Nested coil design

Matching Section Design Goals

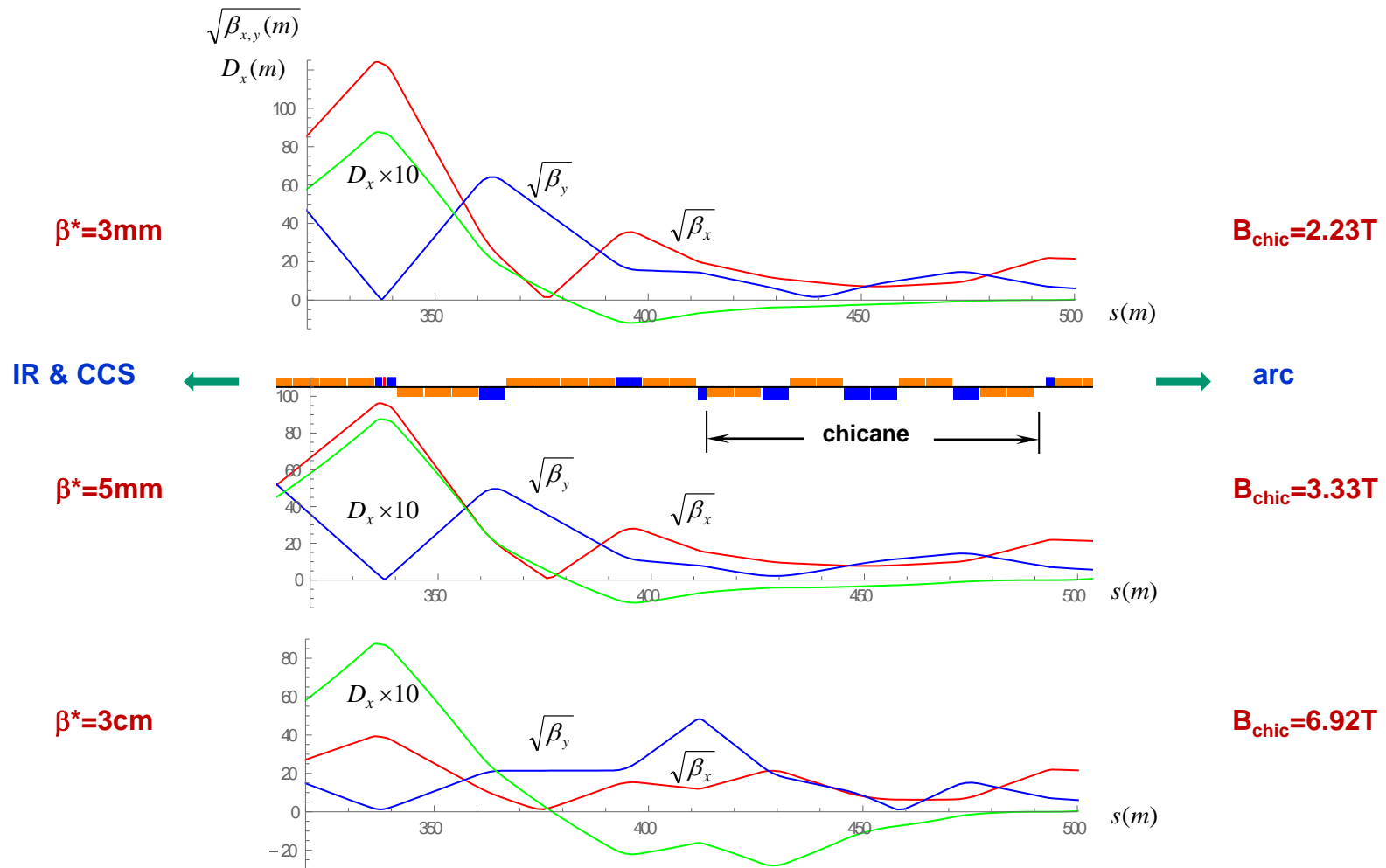
- Design IR-to-Arc matching / RF section which:
 - a) allows for β^* variation in wide range (3mm – 3cm)
 - b) has enough space with low β 's and Dx for RF
 - c) has no straights w/o bending field to spread v 's – all quads are combined-function magnets
 - d) has a place with high β_x and low Dx for halo extraction (we can put special insertions in the arcs but this will increase C – higher costs, lower Lumi)

Conditions a) and c) are difficult to reconcile:

- if β_x changes at a bend then Dx will change all over the ring.
- if we try to adjust the bending angles we will change the orbit.

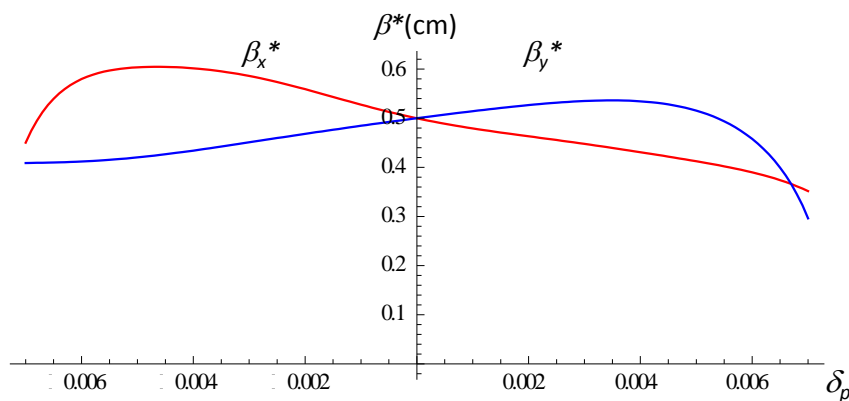
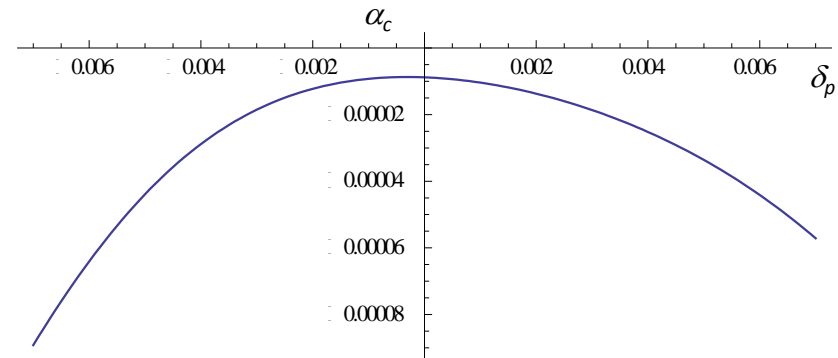
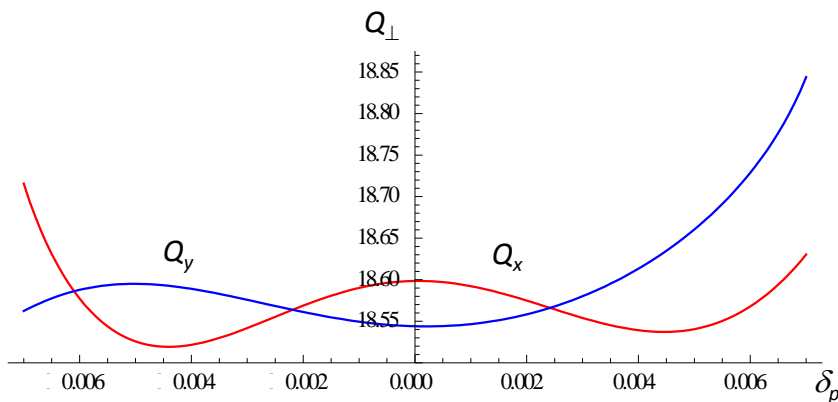
Possible solution: a chicane with variable B-field – no net bending angle, negligible variation in circumference (hopefully)

Matching Section



B-field in chicane is rather low, still it will require mechanical movement of the magnets when changing β^*
 Optics functions at large β^* look ugly (resulting in larger beam size) – further work is necessary!

Momentum acceptance

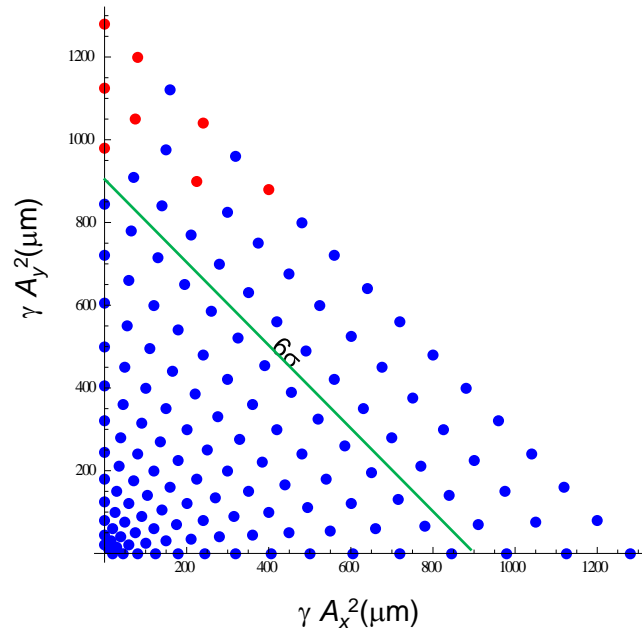


Tunes, beta-functions at IP and the momentum compaction factor α_c vs relative momentum deviation δ_p for $\beta^*=5\text{mm}$.

Due to the possibility to control $d\alpha_c/d\delta_p$ the momentum compaction factor α_c can be made very small w/o compromising the momentum acceptance.

It is not clear, however, how robust it is w.r.t. errors.

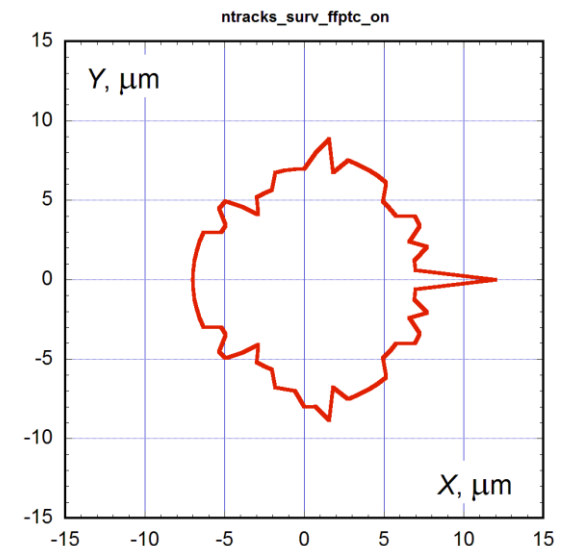
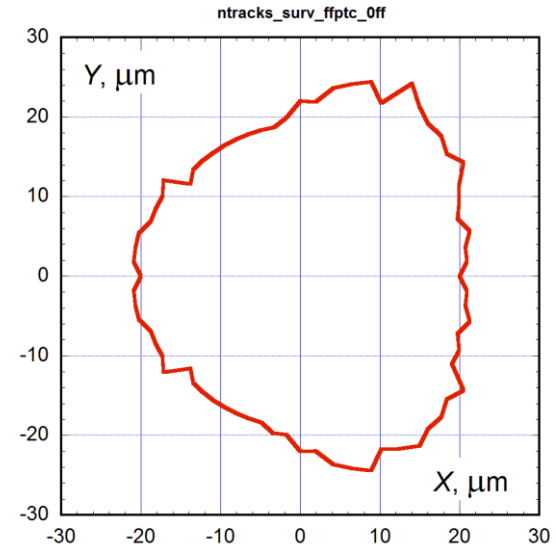
Dynamic Aperture



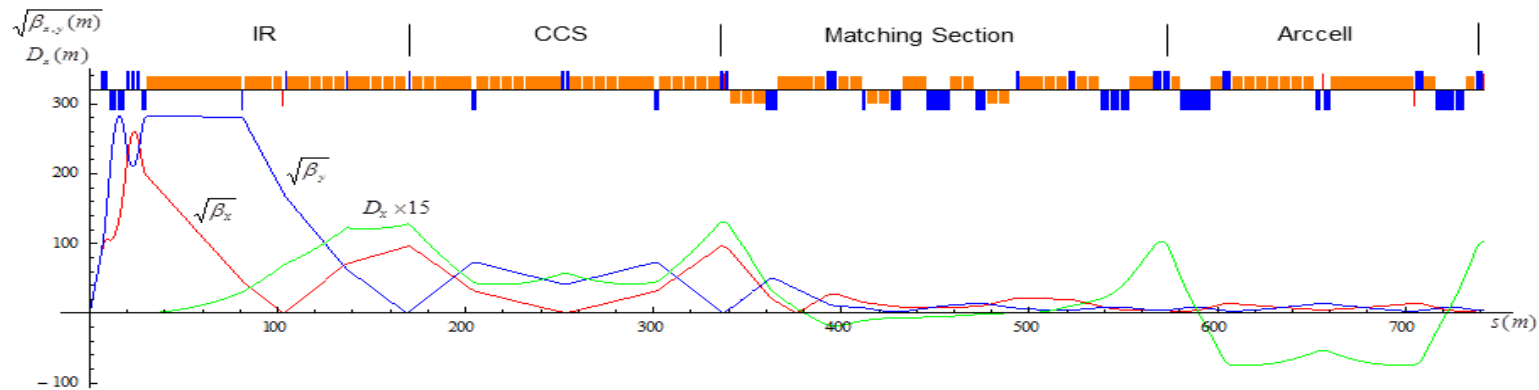
1024 turns on-momentum dynamic aperture at $\beta^*=5$ mm. Left: MAD8 LIE4, right: MADX PTC w/o fringe field (top) and with uncorrected fringe field (bottom).

For nominal parameters $\sigma_{\perp}^*=3\mu\text{m}$.

Previous experience showed that the fringe field effect can be almost completely corrected with dedicated multipole correctors.



Lattice Parameters



Optics functions from IP to the end of the first arc cell (6 such cells / arc) for $\beta^*=5\text{mm}$

3TeV MC lattice parameters	
Beam energy, TeV	1.5
Circumference, km	4.34
Number of IPs	2
β^* , cm	0.5 (0.3-5.0)
Momentum compaction factor, 10^{-5}	-0.88
Stable momentum range	$\pm 0.7\%$
Betatron tunes	18.60/18.54
Dynamic aperture for $\varepsilon_{\perp N}=25\mu\text{m}$	6σ
RF voltage at 1.3 GHz, MV	85
Synchrotron tune	0.0012

Summary

- The design meets all goals promising luminosity $4.5e34$ at $\beta^*=5\text{mm}$.
- It can be improved – β^* -tuning section requires rather high B at $\beta^*>1\text{cm}$
- No fringe-field compensation attempted yet – is not expected to be a problem
- No halo extraction section – the hope is that with pre-collimated beam bent crystals will be enough
- No study of effect of random errors and misalignments, especially on momentum compaction factor – actually a question of critical importance